Storage-Aided Service Surcharge Design for EV Charging Stations

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***Abstract*—The transportation sector is one of the main consumers of global energy. So, its electrification is crucial for a sustainable future. However, the slow developments in the public infrastructure can be a major bottleneck for such electrification. An increasing number of electric vehicle (EV) charging stations are being built across the world to improve this infrastructure. Competition amongst the EV charging stations improves the market efficiency. In this paper, the effect of this competition on the setting of the service surcharge is investigated. The service surcharge design characterization at the Nash equilibrium for both the symmetric as well as the general market conditions is discussed. The value of the storage system to the transportation sector electrification is also analyzed. It is observed that the storage system helps in improving social welfare by reducing the service surcharge in the market, without hurting the revenue of the EV charging stations.**

I. Introduction

With the increasing awareness of global warming, recent years have witnessed rapid electrification in the transportation sector. However, such electrification is challenging for both the end-users and the public infrastructure. Many electric vehicle (EV) owners are suffering from mileage anxiety since there are limited charging stations even in major cities. Their numbers are further low along the highway and other places. The diverse charging technologies and standards make the situation even worse. While electrification may relieve the environmental stress, the associated huge demand for electricity warrants an urgent upgrade of the existing distribution network. To accommodate these challenges, an ideal charging station with diverse charging facilities to serve all the end-users is needed. These stations may also have to upgrade their distribution network. Many charging stations choose to transfer such costs, together with other operational costs, to the end-users as the service surcharges on top of the energy cost. In this work, the competition between the various EV charging stations is investigated in terms of the service surcharges.

In recent years, EV charging price design has been well investigated. Researchers mainly follow the classical cost-benefit analysis framework to design the global optimum price with different optimization targets, such as social welfare [1], load balancing [2], valley filling [3], congestion alleviation [4], and charging station's profit [5]. Some works also examine the price-based demand response using game-theoretical analysis [6]–[8]. However, global optimum price design is always impractical due to the competition amongst the charging stations. Therefore, the designed price can hardly reflect the structure of service surcharge in the real world.

Some works have analyzed the various competitive processes adopted by the EV charging stations. Some of the examples for the competitive processes are citing competitions [9] and energy trading competitions [10]. This paper is based on the pricing competition. The hierarchical game-based approach has been used for both the charging stations and the EVs for the pricing competition [11]. Also, there are studies based on the supermodular game [12] and the Stackelberg game [13], but most of them do not characterize the specific form of the Nash Equilibrium. The energy storage of the EV charging stations has a major impact on the price. Recent works mainly focus on price design [5] and storage control [14]. However, to the best knowledge of the authors, none of the existing works consider the effect of storage on the pricing games.

In this paper, an attempt is made to analytically understand the service surcharge design from a game-theoretic perspective and to investigate the quantitative impact of storage on pricing. All the necessary proofs are presented in the Appendix.

II. System Model

A region having several charging stations and several EVs is considered. For simplicity, it is assumed that the EVs can only be charged at the charging stations within the region, and charging stations will purchase the electricity from the grid to meet the EVs' demand. Specifically, the charging stations are allowed to freely set the charging price, the sum of electricity price from the grid, and the service surcharge. In this section, the local electricity market, the EV model, and the charging station model are introduced in the following subsections.

A. Local Electricity Market

The local utility company is assumed to offer a two-tier Time-of-Use (ToU) pricing scheme, including two continuous periods in a day, i.e., the peak and the off-peak periods. The corresponding electricity prices are fixed as π_h and π_l ($\pi_h \geq \pi_l$), respectively. This information is assumed to be available to all the charging stations. This setting makes it easier to analyze the nature of the service surcharge structure.

B. EV Model

The EV's utility of choosing to be charged at station k. is first characterized. Different charging stations attract users differently. For example, a more convenient location, a lower price, or a better service efficiency may make some charging

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stations more preferable to the users than others. The relative utility of station k is denoted by U_k.

The Logit model [15] is one of the most widely adopted discrete choice models. It can well capture the essence of stochastic behaviors of EV users in choosing the charging stations with different relative utilities. Specifically, under the utility set, $\mathcal{U} = \{U_1, U_2, ..., U_n\}$ and Logit model, the probability of choosing station k is as follows:

$$f_k(U_k) = \frac{e^{\frac{U_k}{\mu}}}{\sum_{i=1}^{n} e^{\frac{U_i}{\mu}}}, \tag{1}$$

where μ reflects the degree of rationality in choosing the charging stations. Intuitively, when $\mu \to 0$, users choose the station with the highest utility. On the other hand, while $\mu \to \infty$, they choose the charging station with an equal probability.

C. EV Charging Station Model

Charging stations make a profit by serving the EVs. Hence, their decisions and revenues are affected by the EVs' aggregate charging demand, which varies across different periods. We denote the charging demand at the peak and at the off-peak periods using the random variables X and Y, their probability density functions are given by $f_x(X), f_y(Y)$, and their cumulative density functions are $F_x(X), F_y(Y)$. Such statistical information can be inferred from the historical data.

Charging stations compete to attract EVs by setting the charging prices, which will directly influence the EV's utility. The EV's utility is assumed to be of the following linear form:

$$U_k(p) = L_k - p, \tag{2}$$

where p denotes the charging price, and L_k denotes the k^{th} station's intrinsic utility besides its price. Note that, the relative utility L_k can also be estimated from the historical data. This model is chosen for more insights in the subsequent analysis.

Finally, it is assumed that the charging stations are informed of the demand statistics (e.g., μ, X, and Y), and they are also aware of each station's relative utility (L_k).

III. Service Surcharge Design

It is assumed that the charging stations compete by setting their charging prices. Since the decision-making in the two periods is completely decoupled, the peak period is analyzed in this section. Subsequently, the results for the off-peak periods are directly obtained. Following the Logit model, given all the charging prices of the n stations $\mathcal{P} = \{p_1, p_2, ..., p_n\}$, station k's expected attracted demand P_k during the peak period is as follows:

$$P_k = \mathbb{E}[X] \frac{e^{\frac{U_k(p_k)}{\mu}}}{\sum_{i=1}^{n} e^{\frac{U_i(p_i)}{\mu}}}, \forall k, \tag{3}$$

where $\mathbb{E}[X]$ denotes the expected total demand, and the fraction denotes the market share. This helps us to characterize the corresponding revenue R_k for each charging station k:

$$R_k = (p_k - \pi_h) P_k, \forall k. \tag{4}$$

It is clear that the revenue for each charging station k not only depends on its own pricing decision (p_k), but also on the pricing decision of the other charging stations. Adopting the revenue function as the utility function for each charging station leads to a game among charging stations.

Formally, we have the following game:

Service Surcharge Design Game (SSDG):

- ***Players***: All the charging stations;
- ***Strategy Space***: The collection of all stations' charging prices. Specifically, $p_k > \pi, \forall k$, where π is the electricity rate set by the local grid operator;
- ***Utility Functions***: The expected revenue R_k of each charging station k defined in (4).

A. Symmetric Market Analysis

The first cut understanding is obtained by analyzing the Nash equilibrium (N.E.) of SSDG in the symmetric market conditions, where all the charging stations share the same attractiveness to all the EVs, i.e., all the L_k's are the same. In this case, we can prove the following Theorem.

Theorem 1: The SSDG in the symmetric market conditions yields a unique N.E.. At the equilibrium, all the charging stations set their charging prices as follows:

$$p_k = \pi_h + \frac{n}{n-1}\mu, \forall k. \tag{5}$$

It is evident that in this case, all the charging stations will charge the same service surcharge $\frac{n}{n-1}\mu$ on top of the electricity price π_h. This explains the physical meaning of μ. It serves as the baseline of the service surcharge, the lower bound of which is exactly μ. This is due to the nature of EV's choice model and is irrelevant to the level of competition.

Although there is a lower bound for the service surcharge, the revenue of each charging station diminishes as the competition becomes fiercer. Specifically, for each charging station k, its revenue during peak period is

$$R_k = \frac{\mu}{n-1}\mathbb{E}[X], \forall k. \tag{6}$$

For the results during the off-peak periods, the only difference is the replacement of the ToU price and the random demand. π_h can be simply replaced with π_l, and X with Y. All the analysis follows the same routine.

B. General Market Conditions

In a more general market condition with heterogeneous L_k's, the N.E. does not enjoy a neat expression as in (5). Still, we can show the N.E. uniquely exists:

Theorem 2: The general SSDG admits a unique N.E.. At the equilibrium, the charging stations price strategies $P^* = \{p_1^*, ..., p_n^*\}$ form the solution to the following equations:

$$\frac{(p_k^* - \pi_h)e^{-\frac{p_k^*}{\mu}}}{p_k^* - \pi_h - \mu} = \sum_{i=1}^{n} e^{\frac{L_i - L_k}{\mu}} e^{-\frac{p_i^*}{\mu}}, \forall k. \tag{7}$$

and the equilibrium charging price p_k^* has a uniform lower bound $\pi_h + \mu$.

According to Eq. (7), we can observe that its left-hand side is decreasing monotonically with respect to p_k when $p_k \geq \pi_h + \mu$. Hence, price p_k has a positive correlation with L_k, which means more attractive charging stations are in a favorable position to set higher charging prices. This also corresponds to our intuition.

The expected revenue of station k throughout the day can be obtained as follows:

$$R_k = (\mathbb{E}[X] + \mathbb{E}[Y])(p_k - \pi_h - \mu). \tag{8}$$

This is remarkable as it reveals that in the competitive environment, to ensure a positive revenue, it is necessary to set the service surcharge to be at least μ.

While we cannot characterize the equilibrium prices in the closed-form expression, one straightforward solution is to utilize the best response dynamics. Specifically, given the initial prices, denote $p_k^{(t)}$ as the t^{th} price response of station k. The classical best response dynamics iterate as follows:

$$\frac{\mu e^{\frac{L_k - p_k^{(t+1)}}{\mu}}}{p_k^{(t+1)} - \pi_h - \mu} = \sum_{i \neq k} e^{\frac{L_i - p_i^{(t)}}{\mu}}, \forall k. \tag{9}$$

However, in SSDG, the best response dynamics are not guaranteed to converge. We prove in the following theorem that a slight modification could help.

Theorem 3: The following response dynamics of general SSDG will converge to the Nash Equilibrium.

$$\frac{(p_k^{(t+1)} - \pi_h) e^{\frac{L_k - p_k^{(t+1)}}{\mu}}}{p_k^{(t+1)} - \pi_h - \mu} = \sum_{i=1}^{n} e^{\frac{L_i - p_i^{(t)}}{\mu}}, \forall k. \tag{10}$$

IV. Value of Storage

The storage system could potentially help improve social welfare by arbitraging against the ToU. In this section, we will again first highlight the structure of service surcharge in the symmetric market condition, and then generalize the result to a more practical setting.

Denote π_s the daily amortized investment and maintenance cost over the lifespan of the storage system. For simplicity, we assume the storage system can be perfectly charged and discharged. We adopt the greedy control policy, which fully charges the storage system during the off-peak period and then first uses the energy in the storage during the peak period. This helps us to characterize the expected utility for each charging station k under the Logit choice model. Denote the peak charging prices for n charging stations by $P_1 = \{p_{11}, ..., p_{1n}\}$, the corresponding off-peak charging prices by $P_2 = \{p_{21}, ..., p_{2n}\}$, and the corresponding storage investment decisions by $\mathcal{C} = \{C_1, ..., C_n\}$. The expected revenue R_k for charging station k can be obtained as follows:

$$\begin{aligned} R_k =& \mathbb{E}[f_k(p_{1k}) X p_{1k}] + \mathbb{E}[f_k(p_{2k}) Y p_{2k}] \\ & - \pi_s C_k - \mathbb{E}[\pi_h (f_k(p_{1k})X - C_k)^+] \\ & - \mathbb{E}[\pi_l f_k(p_{2k}) Y + \pi_l \min\{C_k, f_k(p_{1k})X\}], \end{aligned} \tag{11}$$

where $f_k(p_{1k})$, $f_k(p_{2k})$ denote the market share during peak and off-peak periods.

Different from our analysis without storage, the decision variables are all temporally coupled. In fact, our game-theoretic analysis involves two-stage decision-making. In the first stage the charging stations decide their installed storage capacities and in the second stage they set their charging prices (for different periods) to attract the EVs. Formally, we have:

Storage Decision Game (SDG):

- ***Players***: All the charging stations;
- ***Strategy Space***: The collection of all stations' storage capacity, $C_k > 0, \forall k$;
- ***Utility Functions***: The expected revenue R_k of each charging station k as defined in (11).

Service Surcharge Design Game with Storage (SSDGS):

- ***Players***: All the charging stations;
- ***Strategy Space***: The collection of all stations' peak period charging prices p_{k1} and off-peak period charging prices p_{k2}, $p_{k1}, p_{k2} > \pi_l$;
- ***Utility Functions***: The expected revenue R_k of each charging station k as defined in (11).

Note that, the utility function is the same for both the games. However, the decision variables (strategy spaces) are different. To analyze the two-stage game, the notion of backward induction is used. The equilibrium of SSDGS is characterized, which is subsequently used as the subgame perfect equilibrium to understand the SDG.

A. Symmetric Market Analysis

The symmetric market condition is analyzed for having a better intuition. This results in the following theorem:

Theorem 4: The two-stage game in the symmetric market condition admits a unique N.E.. At N.E., the charging price p_1^*, the off-peak charging price p_2^*, and the storage investment C^* can be characterized as follows:

$$p_1^* = \pi_h + \frac{n}{n-1}\mu - \frac{(\pi_h - \pi_l)\mathbb{E}[X | X \leq nC]}{\mathbb{E}[X]}, \tag{12}$$

$$p_2^* = \pi_l + \frac{n}{n-1}\mu, \quad C^* = \frac{1}{n} F_x^{-1}\left(\frac{\pi_h - \pi_l - \pi_s}{\pi_h - \pi_l}\right). \tag{13}$$

Note that the equilibrium charging price during off-peak remains unchanged, as compared to the case without storage. Only the charging price during peak period is reduced. The storage investment capacity at the equilibrium coincides with the results in the classical newsvendor problem [16].

In this case, it can be observed that the revenue for each charging station k remains the same. This corroborates the fact that the storage system helps in reducing the service surcharge in the market and it transfers all the arbitrage gains to the end EV users. In such cases, the storage system should be heavily subsidized to the charging stations.

B. General Market Conditions

We can further characterize the N.E. in the general market conditions as follows:

Theorem 5: The two-stage game admits a unique N.E.. At the equilibrium, the charging stations peak price strategies $P_1^* =$

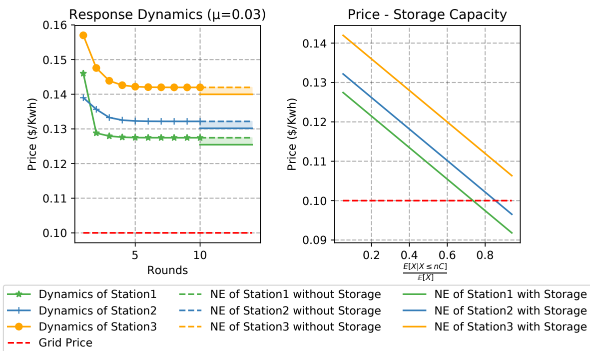

Fig. 1: Nash Equilibrium of SSDGS.

$\{p_{11}^*,...,p_{1n}^*\}$, off-peak price strategies $P_2^* = \{p_{21}^*,...,p_{2n}^*\}$ and storage policy $\mathcal{C}^* = \{C_1^*,...,C_n^*\}$ satisfy the following equations:

$$f_k(p_{1k}^*)+(p_{1k}^*-(\pi_h-\alpha(\pi_h-\pi_l)))\, f_k'(p_{1k}^*)=0, \forall k, \quad (14)$$

$$\frac{(p_{2k}^*-\pi_h)e^{-\frac{p_{2k}^*}{\mu}}}{p_{2k}^*-\pi_h-\mu}=\sum_{i=1}^{n}e^{\frac{L_i-L_k}{\mu}}e^{-\frac{p_{2i}^*}{\mu}}, \forall k, \quad (15)$$

$$C_k^*=f_k(p_{1k}^*)F_x^{-1}(\frac{\pi_h-\pi_l-\pi_s}{\pi_h-\pi_l}), \forall k, \quad (16)$$

$$\alpha=\frac{\mathbb{E}[X|X\leq F_x^{-1}(\frac{\pi_h-\pi_l-\pi_s}{\pi_h-\pi_l})]}{\mathbb{E}[X]}. \quad (17)$$

Proposition 1: The following response dynamics of the general SSDGS will converge to the N.E..

$$\frac{(p_k^{(t+1)}-\pi)e^{\frac{L_k-p_k^{(t+1)}}{\mu}}}{p_k^{(t+1)}-\pi-\mu}=\sum_{i=1}^{n}e^{\frac{L_i-p^{(t)}}{\mu}}, \forall k, \quad (18)$$

where $\pi=\pi_h-\alpha(\pi_h-\pi_l)$.

Compared to (10), the dynamics given in (18) just change π_h to another fixed value π. We use this dynamics to numerically characterize the equilibrium. As indicated in Fig. 1, considering a 3-charging station case, the storage system again reduces the charging prices during peak period, and these prices monotonically converge in 10 iterations. When the storage capacity is increased, the peak charging equilibrium price decreases linearly and it falls below the peak price offered by the grid, which strongly reflects the storage system's ability to transfer the arbitrage gains into social welfare.

V. Numeric Studies

In this section, the real-world pricing considering service capacity and geographical information are characterized. Also, for this general setting, the value of investing in storage and service capacity is demonstrated.

A. Practical Models

Unlike the model used in the above theoretical analysis, charging stations in the real world have limited service capacities, and their attractiveness to the users varies geographically.

The $tanh$ function is used to characterize the limited service capacity of the charging station. Formally, denote the service capacities of the charging stations by $S=\{s_1,s_2,...,s_n\}$. When the market share of charging station k is M_k, its service volume V_k is $s_k tanh\left(\frac{M_k}{s_k}\right), \forall k$.

Besides, users have a geographical preference for choosing a charging station. For example, the distance to charging stations can be a crucial factor that affects the user preferences L_k's.

Specifically, the users are divided into m groups based on their geographic locations as $G=\{g_1,g_2,...,g_m\}$. Follow the hoteling model [17], and the relative utility of charging station k on user group j is defined as follows:

$$L_{k,j}=L_k-\theta_j d_{k,j}^2, \forall k, j, \quad (19)$$

where θ_j denotes group j's sensitivity to the distance; μ_j denotes group j's rationality to utility; and $d_{k,j}$ denotes the Euclidean distance between group j and charging station k. Hence, station k's revenue R_k is:

$$R_k=s_k tanh\left(\frac{M_k}{s_k}\right)(p_k-\pi_h), \forall k, \quad (20)$$

$$M_k=\sum_{j=1}^{m}A_j\frac{e^{\frac{L_{k,j}-p_k}{\mu_j}}}{\sum_{i=1}^{n}e^{\frac{L_{i,j}-p_i}{\mu_j}}}, \forall k, \quad (21)$$

where A_j denotes group j's demand for charging.

B. Value of Service Capacity

We adopt the pricing scheme in Beijing, China. In the two-tier ToU, the peak period is from 10:00 am to 10:00 pm, with π_h to be \$0.10/kWh, and π_l to be \$0.06/kWh. The Pecan Street dataset is used to simulate EV's hourly charging demand [18].

Consider a rectangular region with 6 charging stations, there are 900 groups of EVs that are randomly distributed over this area with different charging demand, different distance sensitivity θ and different rationality μ. Stations' relative revenues are set to be $\$0.09, \$0.118, \$0.108, \$0.106, \$0.112, \$0.104$, and the ratios of their service capacity to the average regional demand are $0.4, 0.3, 0.3, 0.2, 0.5, 0.7$.

First, the value of service capacity is highlighted and a control group is set up, which doubles the service capacity of the third charging station. Fig. 2 illustrates the geographical distribution of the users based on their different preferences. The small circles having the same color represent an EV user group that favours a particular station. When the third charging station's service capacity is doubled, it attracts significantly more users and its "control region" is expanded further.

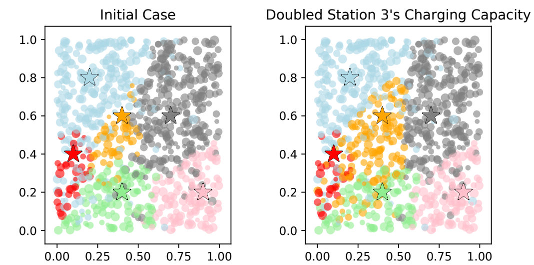

Fig. 2: Geographical Distribution of Preferences.

TABLE I: Market Analysis

Station	Initial Case						Doubled Station 3's Service Capacity					
	1	2	3	4	5	6	1	2	3	4	5	6
Revenue Share (%)	10.08	20.37	**18.88**	14.42	22.71	13.53	9.69	19.88	**21.06**	14.16	22.07	13.15
Market Share (%)	11.87	18.97	**19.29**	14.01	21.69	14.17	11.29	18.50	**22.07**	13.65	20.88	12.60
Surcharges (c/KWh)	3.74	5.20	**4.74**	5.03	4.65	4.06	3.68	5.05	**4.10**	4.91	4.56	4.00

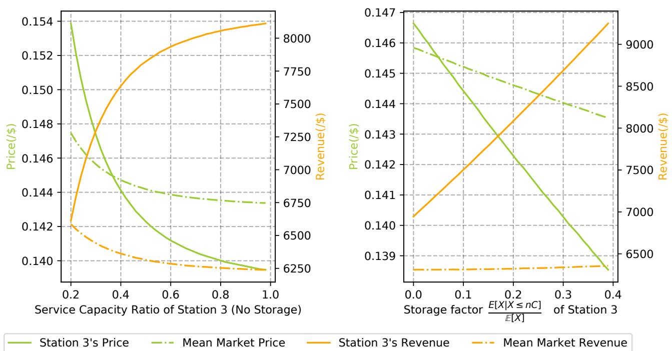

Fig. 3: Price and Revenue Analysis.

TABLE. I illustrates the market share and the associated revenue. It can be observed that increasing the third charging station's capacity significantly changes its pricing strategy: the service surcharge has reduced by 13.51%, but the revenue share has increased by 11.55%. While service surcharges of other charging stations have dropped by 1% to 2%. Hence, investment to improve the service capacity enables the users to avail services timely, and it also improves the social welfare.

C. Value of Storage

Fig. 3 illustrates two relationships. The first subfigure indicates the changes in the market price and the changes in the revenue with respect to the service capacity of the third station. The second subfigure studies their variation with respect to the storage capacity of the third station. Both measures are effective in improving the revenue of the station and in reducing the average market price. However, the first measure reduces the average revenue of all the charging stations. On the contrary, investing in the storage capacity offers charging stations the ability to reduce costs by arbitrage against ToU pricing, and introduces extra revenue to the market. The effect of storage capacity on the revenue will not be saturated, and almost all the additional profits are converted into social welfare, which is also consistent with the previous analysis. This suggests that the investment in increasing the storage capacity is a benign social welfare transfer mechanism.

VI. Concluding Remarks

In this work, the EV charging station's decision-making on the service surcharge in different competitive scenarios was investigated, including symmetric and general market conditions. The value of storage to the EV end-users, EV charging stations has been examined. In the symmetric market condition, the storage system transfers all the arbitrage gains to the end-users as improved social welfare, which highlights its value to be adopted as a public asset by the system operator.

This work can be extended in various ways. For example, it would be interesting to consider a more practical dynamic pricing schemes. This may incur significant challenges for the analysis on the value of storage, as the greedy control policy may not be optimal in the general pricing schemes. Also, the work makes an implicit assumption that the EV charging demands over time follow the same distribution. Machine learning techniques may improve the quality of EV charging station's decision-making by relaxing such an assumption.

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APPENDIX

A. Proof for Theorem 1

It suffices to solve the system of equations, consisting of the first order optimality condition for each charging station, which, after simplicity, is as follows:

$$(\pi_h + \mu - p_k) \sum_{i=1}^{n} e^{-\frac{p_i}{\mu}} + (p_k - \mu) e^{-\frac{p_k}{\mu}} = 0, \forall k. \quad (22)$$

In the symmetric market condition, all the charging stations will set the same charging price:

$$p_k = \pi_h + \frac{n}{n-1}\mu, \forall k. \quad (23)$$

It's also straightforward to verify that the second order optimality condition at the equilibrium is strictly negative:

$$\left.\frac{\partial^2 R_k}{\partial {p_k}^2}\right|_{p_k = \pi_h + \frac{n}{n-1}\mu} = -\frac{1}{n\mu} < 0, \forall k. \quad (24)$$

Hence p_k^* forms the unique equilibrium. ■

B. Proof for Theorem 2

Similar to the proof for Theorem 1, the first order optimality condition is as follows:

$$\frac{(p_k - \pi_h) e^{-\frac{p_k}{\mu}}}{p_k - \pi_h - \mu} = \sum_{i=1}^{n} e^{\frac{L_i - L_k}{\mu}} e^{-\frac{p_i}{\mu}}, \forall k. \quad (25)$$

Then we prove the uniqueness of the solution to Eqs. (25). Define $G(p_k)$ and M as follows:

$$G(p_k) = \frac{(p_k - \pi_h) e^{-\frac{p_k}{\mu}}}{p_k - \pi_h - \mu}, \quad M = \sum_{i=1}^{n} e^{\frac{L_i}{\mu}} e^{-\frac{p_i}{\mu}}. \quad (26)$$

From Eqs. (25), p_k can be represented by $G^{-1}(Me^{-\frac{L_k}{\mu}})$. According to Eq. (26), we have:

$$\sum_{i=1}^{n} \frac{e^{-\frac{G^{-1}(Me^{-\frac{L_i}{\mu}})}{\mu}}}{Me^{-\frac{L_i}{\mu}}} = 1. \quad (27)$$

Note that, if we consider the left hand side of (27) as M's function, denoted by $T(M)$, then we can show its first order derivative is always negative. Hence, $T(M)$ is monotonically decreasing. It indicates that, in Eqs. (25), the right-hand side is positive, so p_k^* should be either lower than π_h or higher than $\pi_h + \mu$. Meanwhile, to achieve positive revenue guarantees $p_k^* > \pi_h + \mu, \forall k$. Hence, $T(M)$ can be rearranged as follows:

$$T(M) = \sum_{i=1}^{n} \frac{p_i^* - \pi_h - \mu}{p_i^* - \pi_h}. \quad (28)$$

Note that when M is small enough, $p_k^* \to \infty, \forall k$, and hence $T(M) \to n$. On the other hand, when M is large, $p_k^* \to \pi_h + \mu, \forall k$, and hence $T(M) \to 0$. Therefore, a unique solution M^* to $T(M) = 1$ exists.

After checking the second order optimality condition at the equilibrium, we can conclude our proof. ■

C. Proof for Theorem 3

The current price response of each charging station ($p_k^{(t+1)}$) depends on the previous price response of all charging stations ($p_i^{(t)}$'s). Define $S^{(t)} = \sum_{i=1}^{n} e^{\frac{L_i - p_i^{(t)}}{\mu}}$. Given the initial price $P^{(1)} = \{p_1^{(1)}, p_2^{(1)}, ..., p_n^{(1)}\}$, define $P^{(t)} = \{p_1^{(t)}, p_2^{(t)}, ..., p_n^{(t)}\}$, $S^{(t)} > S^{(t-1)}$ for round t $(t > 1)$. Note that $p_k^{(t+1)} > \pi_h + \mu$, which guarantees the monotonicity of the left term:

$$\frac{\partial \frac{(p_k^{(t+1)} - \pi_h) e^{\frac{L_k - p_k^{(t+1)}}{\mu}}}{p_k^{(t+1)} - \pi_h - \mu}}{\partial p_k^{(t+1)}} < 0, \forall k. \quad (29)$$

It's straightforward to verify $p_k^{(t+1)} < p_k^{(t)}$ and $S^{(t+1)} > S^{(t)}$. Hence, if $S^{(2)} > S^{(1)}$, $p_k^{(t)}$ will decrease monotonically over t when $t > 1$.

As $S(t)$ is positive for any t, and $p_k^{(t)} > \pi_h + \mu, \forall t$, according to Monotone Convergence Theorem, $p_k^{(t)}$ will converge to a unique limit. If $S^{(2)} \leq S^{(1)}$, we can obtain similar results. Because of the uniqueness of Nash equilibrium, this response dynamics will converge to this unique N.E.. ■

D. Proof for Theorem 4

Denote the charging stations' equilibrium peak price strategies by $P_1^* = \{p_{11}^*, ..., p_{1n}^*\}$, off-peak price strategies by $P_2^* = \{p_{21}^*, ..., p_{2n}^*\}$ and storage investment by $\mathcal{C}^* = \{C_1^*, ..., C_n^*\}$. The first order optimality condition with respect to P_1^*, P_2^* and $\mathcal{C}^*$ requires for all k:

$$f_k(p_{1k}^*) + \left(p_{1k}^* - \pi_h + \frac{H[X](\pi_h - \pi_l)}{\mathbb{E}[X]}\right) f_k'(p_{1k}^*) = 0, \quad (30)$$

$$f_k(p_{2k}^*) + (p_{2k}^* - \pi_l) f_k'(p_{2k}^*) = 0, \quad (31)$$

$$C_k^* = f_k(p_{1k}^*) F_x^{-1}(\frac{\pi_h - \pi_l - \pi_s}{\pi_h - \pi_l}), \quad (32)$$

where

$$H[X] = E\left[X \middle| X \leq F_x^{-1}\left(\frac{\pi_h - \pi_l - \pi_s}{\pi_h - \pi_l}\right)\right]. \quad (33)$$

The symmetric market condition simplifies solving the system of equations. It's straightforward to show the equilibrium is exactly as characterized by the theorem.

The uniqueness of p_1^* and p_2^* can be proved following the same routine in Appendix A. Additionally, the storage capacity C^* is uniquely decided by p_1^*, which automatically guarantees its uniqueness. This concludes our proof. ■

E. Proof for Theorem 5

Again, we observe exactly the same set of first order optimality conditions in Appendix D. The difficulty is that we don't have the symmetric market conditions.

Note that after replacing the general market share $f_k(p)$ with its definition, combining Eqs. (30) and (31) leads to a similar form as Eqs. (25). Hence, the uniqueness of equilibrium peak and off-peak price strategy P_1^* and P_2^* can be proved following the same routine in Appendix B. Same as we proceed in Appendix D, the storage investment $\mathcal{C}^*$ is again uniquely decided by P_1^*. Therefore its uniqueness automatically holds given the uniqueness of P_1^*. This concludes our proof. ■